stability of current sheet with normal component of magnetic field and plasma flows

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• Magnetic reconnection—*explosive energy release*

**CME and solar flare**

*Christe+, 2017*

**Substorm**

*Eastwood+, 2017*
What triggers the fast magnetic reconnection at large scales?
Onset of reconnection

- Need to break the “frozen-in” condition
- Generalized Ohm’s law:

\[
E + u \times B = \eta J + \frac{J \times B}{ne} - \frac{\nabla \cdot P_e}{ne} + \frac{m_e}{ne^2} \frac{\partial J}{\partial t}
\]

- Resistivity
- Hall
- Electron pressure
- Electron inertia

Steady reconnection model in resistive-MHD

\[
S = \frac{LV_a}{\eta} > 10^8
\]

- Lundquist number (magnetic Reynolds number)
- \( a \sim S^{-1/2} \)
- \( \frac{u_i}{V_a} \sim S^{-1/2} \)
### Typical parameters of heliospheric plasmas

<table>
<thead>
<tr>
<th>Location</th>
<th>Plasma</th>
<th>Size (m)</th>
<th>$T_e$ (eV)</th>
<th>$n_e$ (m$^{-3}$)</th>
<th>$B_T$ (Tesla)</th>
<th>$S$</th>
<th>$\lambda$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar system</td>
<td>Magnetopause$^{81}$</td>
<td>$6 \times 10^7$</td>
<td>300</td>
<td>$1 \times 10^7$</td>
<td>$5 \times 10^{-8}$</td>
<td>$6 \times 10^{13}$</td>
<td>$9 \times 10^2$</td>
<td>$B_R = B_T$ (p. 267)</td>
</tr>
<tr>
<td></td>
<td>Magnetotail$^{81}$</td>
<td>$6 \times 10^8$</td>
<td>600</td>
<td>$3 \times 10^5$</td>
<td>$2 \times 10^{-8}$</td>
<td>$4 \times 10^{15}$</td>
<td>$1.3 \times 10^3$</td>
<td>$B_R = B_T, T_i = 4.2$ keV (p. 233) (p. 92)</td>
</tr>
<tr>
<td></td>
<td>Solar wind$^{81}$</td>
<td>$2 \times 10^{10}$</td>
<td>10</td>
<td>$7 \times 10^6$</td>
<td>$7 \times 10^{-9}$</td>
<td>$3 \times 10^{12}$</td>
<td>$2 \times 10^5$</td>
<td>(p. 79)</td>
</tr>
<tr>
<td></td>
<td>Solar corona$^{81}$</td>
<td>$1 \times 10^7$</td>
<td>200</td>
<td>$1 \times 10^{15}$</td>
<td>$2 \times 10^{-2}$</td>
<td>$1 \times 10^{13}$</td>
<td>$4 \times 10^7$</td>
<td>Neutral particle effects are weak$^{82}$</td>
</tr>
<tr>
<td></td>
<td>Solar chromosphere$^{82}$</td>
<td>$1 \times 10^7$</td>
<td>0.5</td>
<td>$1 \times 10^{17}$</td>
<td>$2 \times 10^{-2}$</td>
<td>$1 \times 10^{8}$</td>
<td>$3 \times 10^8$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Solar tachocline$^{83,84}$</td>
<td>$1 \times 10^7$</td>
<td>200</td>
<td>$1 \times 10^{29}$</td>
<td>1</td>
<td>$1 \times 10^9$</td>
<td>$5 \times 10^{10}$</td>
<td></td>
</tr>
</tbody>
</table>

**Sweet-Parker current sheet**

Very thin and Extremely Slow!

\[ S = \frac{LV_a}{\eta} > 10^8 \]

\[ \frac{a}{L} \sim S^{-\frac{1}{2}} \]

\[ \frac{u_i}{V_a} \sim S^{-1/2} \]

Lundquist number (magnetic Reynolds number)
Are resistive current sheets stable?

- Tearing instability (Furth+, 1963; etc.)

\[
\gamma_m \tau_A \sim S^{-1/2}
\]

- Magnetic field lines

- \[ \lambda \]

- \[ 2a \]

\[
\begin{align*}
\tau_A &= \frac{a}{V_A} \\
S &= \frac{a V_A}{\eta}
\end{align*}
\]
Aspect ratio (thickness) of the current sheet determines whether fast tearing can happen

\[ \gamma \tau_A \sim S^{\frac{1}{2}} \]

\[ \tau_A = \frac{a}{V_A} \]

\[ S = \frac{aV_A}{\eta} \]

\[ \tau_{AL} = \frac{L}{V_A} \]

\[ S_L = \frac{LV_A}{\eta} \]

\[ \gamma \tau_{AL} \frac{a}{L} \sim S_L^{\frac{1}{2}} \left( \frac{a}{L} \right)^{-\frac{1}{2}} \rightarrow \gamma \tau_{AL} \sim S_L^{\frac{1}{2}} \left( \frac{a}{L} \right)^{-\frac{3}{2}} \]

- \[ \frac{a}{L} > S_L^{-\frac{1}{3}} : \text{extremely slow growth} \]
- \[ \frac{a}{L} < S_L^{-\frac{1}{3}} : \text{extremely fast growth} \]
- \[ \frac{a}{L} = S_L^{-\frac{1}{3}} : \gamma \tau_{AL} \sim O(1) \]

"ideal tearing" (Pucci & Velli, 2013)
Fast recursive tearing in a collapsing CS

Collapse of macroscopic current sheet $\rightarrow$ tearing instability

$\frac{L}{a} \sim S^{\frac{1}{3}}_L$

$\rightarrow$ secondary current sheet $\rightarrow$ dynamic lengthening

$\frac{L}{a} \sim S^{\frac{1}{3}}_L$

$\rightarrow$ tearing instability

$S_{L,n} \sim (S_{L,0})^{\left(\frac{3}{4}\right)^n}$

$L_n \sim L_0(S_{L,0})^{-1+\left(\frac{3}{4}\right)^n}$

Tenerani+, 2015
Helmet streamers

Earth’s magnetotail

Bessho and Bhattacharjee, 2014

Lapenta and Knoll, 2003

Normal component of magnetic field is often non-negligible
Kinetic scale: $B_n$ stabilizes the current sheet

Bessho and Bhattacharjee, 2014
2D PIC simulation
How to maintain equilibrium?

In many cases the pressure gradient is too small to fully balance the Lorentz force. To address this, field-aligned flows can be introduced. These flows mimic off-diagonal pressure terms and help maintain equilibrium.

**Pressure Gradient**

\( \frac{\partial p}{\partial x} \)

**Field-Aligned Flows**

Ionospheric outflow

Mimic off-diagonal pressure
MHD equilibrium in a 1D static Harris current sheet

\[ B = B_0 \tanh \left( \frac{Z}{a} \right) \hat{e}_x + B_z \hat{e}_z \]

\[-\nabla P^T + B \cdot \nabla B / \mu_0 = 0\]

\[ \nabla \times (B \cdot \nabla B) = 0\]

Equilibrium with flow (assume incompressibility)

• One-fluid: Alfvénic flow

\[ \mathbf{V} \cdot \nabla \mathbf{V} = -\frac{1}{\rho} \nabla P^T + \mathbf{V}_A \cdot \nabla \mathbf{V}_A \rightarrow \]

\[ \begin{align*}
V & \equiv V_A \\
\nabla P^T & \equiv 0
\end{align*}\]

\[ V_A = \frac{B}{\sqrt{\mu_0 \rho}}\]
Two-ion model (effective $P_{xz}$)

\[ \rho_\pm \mathbf{V}_\pm \cdot \nabla \mathbf{V}_\pm = -\nabla P_\pm + \frac{q}{m_p} \rho_\pm \left( \mathbf{E} + \frac{1}{c} \mathbf{V}_\pm \times \mathbf{B} \right) \]

\[ 0 \approx -qn \left( \mathbf{E} + \mathbf{V}_e \times \mathbf{B} \right) \]

\[ \rho_+ \mathbf{V}_+ \cdot \nabla \mathbf{V}_+ + \rho_- \mathbf{V}_- \cdot \nabla \mathbf{V}_- = -\nabla \left( P + \frac{B^2}{8\pi} \right) + \frac{1}{4\pi} \mathbf{B} \cdot \nabla \mathbf{B} \]

\[ \mathbf{V} = \frac{1}{2} (\alpha_+ - \alpha_-) \mathbf{V}_A \]

\[ \rho_+ = \rho_- = \frac{1}{2} \rho \]

\[ \mathbf{V}_\pm = \pm \alpha_\pm \mathbf{V}_A \]

Counter-propagating

One-fluid

\[ 0 \leq \text{Net flow speed} \leq V_A \]

\[ \mu = \frac{\mathbf{V}}{V_A} \in [0,1] \]

\[ \alpha_+^2 + \alpha_-^2 = 2 \]
Perturbation equations

\[ f(x, z, t) = f(z) \exp(ikx + \gamma t) \]

momentum equation

\[ \frac{1}{2} \rho_0 (\gamma u_\pm + u_\pm \cdot \nabla V_\pm + V_\pm \cdot \nabla u_\pm) = -\nabla p_\pm + \frac{1}{2} \frac{q}{m_p} \rho_0 \left( E_1 + \frac{1}{c} u_\pm \times B + \frac{1}{c} V_\pm \times b \right) \]

\[ u = \frac{1}{2} (u_+ + u_-), \quad w = \frac{1}{2} (u_+ - u_-) \]

\[ \gamma u + f_- \cdot \nabla V_A + V_A \cdot \nabla f_- = -\frac{1}{\rho_0} \nabla (p_+ + p_-) + \frac{1}{c \rho_0} (j \times B + J \times b) \]

\[ \gamma w + f_+ \cdot \nabla V_A + V_A \cdot \nabla f_+ = -\frac{1}{\rho_0} \nabla (p_+ - p_-) + \frac{q}{m_p c} \left[ w \times B + \frac{1}{2} (\alpha_+ + \alpha_-) V_A \times b \right] \]

\[ f_\pm = (\alpha_+ u_+ \pm \alpha_- u_-) / 2 \]

\[ \frac{a}{d_i} \propto \]

\[ w = \frac{1}{2} (\alpha_+ + \alpha_-) b \]
Induction equation

\[ \gamma b = b \cdot \nabla V - V \cdot \nabla b + B \cdot \nabla u - u \cdot \nabla B + \frac{1}{S} \nabla^2 b \]

\[ S = aV_A / \eta \]

Final equation set to solve:

\[
\gamma (u''_z - k^2 u_z) + \mu \left\{ B_z (u'''_z - k^2 u'_z) + ik \left[B_x (u''_z - k^2 u_z) - B''_x u_z \right] \right\} = \\
\sigma \left\{ B_z (b'''_z - k^2 b'_z) + ik \left[B_x (b''_z - k^2 b_z) - B''_x b_z \right] \right\} \\
\gamma b_z = (ikB_x u_z + B_z u'_z) - \mu (ikB_x b_z + B_z b'_z) + \frac{1}{S} (b''_z - k^2 b_z) \\
\mu = (\alpha_+ - \alpha_-)/2, \quad \sigma = 1 - (\alpha_+ + \alpha_-)^2 / 4
\]
Consider \( B_z = 0, \alpha_{\pm} = 0 \):

\[
\gamma \left( u''_z - k^2 u_z \right) + \mu \left\{ B_z \left( u'''_z - k^2 u'_z \right) + ik \left[ B_x \left( u''_z - k^2 u_z \right) - B''_x u_z \right] \right\} = \sigma \left\{ B_z \left( b'''_z - k^2 b'_z \right) + ik \left[ B_x \left( b''_z - k^2 b_z \right) - B''_x b_z \right] \right\}
\]

\[
\gamma b_z = (ikB_x u_z + B_z u'_z) - \mu (ikB_x b_z + B_z b'_z) + \frac{1}{S} (b''_z - k^2 b_z)
\]

\[
\mu = (\alpha_+ - \alpha_-) / 2, \quad \sigma = 1 - (\alpha_+ + \alpha_-)^2 / 4
\]

Classic tearing mode equation

Eigenvalue (\( \gamma \)) & boundary-value \((u_z, b_z \rightarrow 0 \text{ as } |z| \rightarrow \infty)\) problem
$B_Z = 0$, add (one-fluid) shear flow

$B_Z = 0$, varying $V$, $S = 10^4$

$S = \frac{aV_A}{\eta}$

Maximum growth rate

- Shear flow stabilizes tearing mode (sub-aflvénic regime)
- When $V \equiv V_A$: much more stable compared to sub-aflvénic flow

CS needs to be thinner to trigger fast instability
One fluid with finite $B_z$ stabilizes the current sheet

\[ \gamma \tau_a \sim S^{-1} \]

\[ \mu = V/V_A \]

\[ \gamma \sim \frac{\eta}{a^2} \]

\[ \mu = 1.0 \]

\[ \gamma \sim \frac{\eta}{a^2} \]

$S = 10^4, \mu = 1.00$

\[ \omega \]

\[ Y \]

Maximum growth rate

\[ Y_m \]

\[ 10^{-2} \]

\[ 10^{-3} \]

\[ 10^{-4} \]

\[ 10^{-5} \]

\[ 10^{3} \]

\[ 10^{4} \]

\[ 10^{5} \]

\[ 10^{6} \]
Adjust the net flow speed

In the two-ion model, the faster average flow makes CS more unstable.
Conclusion I

• We studied: **1D current sheet with a constant normal $B$**
  • Magnetic tension force is balanced by the shear stress
  • One-ion: only Alfvénic flow can maintain $O^{th}$-order equilibrium with $B_z$
  • Two-ion: we can control the net flow (parallel to $B_0$) speed to be $[0 – 1] V_A$
    • Alfvénic net flow (essentially one-fluid): the most unstable
  • $B_z$ indeed stabilizes the current sheet
    • Large $B_z$ makes the mode purely diffusive ($\gamma \tau_a \sim S^{-1}$)

\[
\text{1D current sheet with finite } B_z \text{ cannot be more unstable than that without } B_z
\]

What drives the onset of reconnection in finite-$B_n$ current sheets?

Shi+, 2021 JGR
Current sheet with a jet/wake

- Current sheets within turbulent environments (magnetosheath, solar wind, solar corona)
- Current sheet at the tip of helmet streamer

Rappazzo+2010

Bettarini+2006
Configuration of background fields

• Harris-current sheet with constant guide field
• A jet that can have arbitrary angle with the magnetic field
Equation set for perturbations

- Project $V$ and $B$ to $k$

$$\gamma (u''_y - k^2 u_y) + ik \left[ V_x (u''_y - k^2 u_y) - V''_x u_y \right] = k \left[ B_x (b''_y - k^2 b_y) - B''_x b_y \right]$$

$$\gamma b_y = -ik V_x b_y - k B_x u_y + \frac{1}{S} \left( b''_y - k^2 b_y \right)$$
Streaming instabilities – ideal MHD ($\eta = 0$)

$V \parallel B$

Sausage (varicose) mode

Kink (sinuous) mode

Current sheet stabilizes the jet instabilities

Kink mode is more susceptible to the stabilizing effect
Resistive streaming instabilities and transition to tearing mode

\[ \frac{V_0}{B_0} = 2.5 \]

At \( 1 < \frac{V_0}{B_0} < 2 \), the sausage mode is a mixture of tearing and streaming-sausage modes.

At \( \frac{V_0}{B_0} < 1 \), the sausage mode is tearing-like (because the pure streaming mode is totally stable).
Jet increases the growth rate of tearing without changing the scaling relation between $\max(\gamma)$ and $S$ for $\frac{V_0}{B_0} < 1$. 
\( V \perp B \): oblique modes

The system is determined by \( V \) and \( B \) projected on \( k \):

As \( k \) rotates from \( B \) to \( V \), the mode transitions from pure tearing to pure streaming
**$V \perp B + \text{guide field}$**

- Guide field suppresses the oblique tearing mode but does not affect the parallel mode ($k \parallel x$) (Shi+2020)
- Guide field suppresses the streaming instability
- With certain parameters, the fastest growing modes are oblique (peaks in $\max(\gamma) - \theta$ curves)

\[
S = 10^4
\]
Conclusion II

• For $V \parallel B$:
  • Sausage streaming mode couples with tearing mode for $\frac{V_0}{B_0} < 2$
  • For $\frac{V_0}{B_0} < 1$, the sausage mode is tearing-like, the jet increases the growth rate but does not change the scaling relation $\max(\gamma) - S$

• For $V \perp B$:
  • The mode transits from pure tearing to pure streaming as $k$ rotates from parallel to $B$ to parallel to $V$: depending on $V_0/B_0$, the most unstable mode has either $k \parallel B$ or $k \perp B$
  • With a guide field, the fastest growing mode may be oblique for large $V_0/B_0$, because it suppresses the jet instability