Can Field-Particle Correlations be used to Discern the Nature of Solar Wind Heating?

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Open Questions in Characterizing Turbulence

(Kiyani, Osman, & Chapman, 2015)
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Turbulence Studies Focus on:
- Scale dependent nature of nonlinear interactions

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Open Questions in Characterizing Turbulence

Turbulence Studies Focus on:

- Scale dependent nature of nonlinear interactions
- Statistical descriptions of the distribution of mass, momentum, and energy

Osman et al. 2011

Chandran et al. 2014

Howes 2016
Open Questions in Characterizing Turbulence

Richardson & Smith, 2003; Voyager 2

Turbulence Studies Focus on:

- Scale dependent nature of nonlinear interactions
- Statistical descriptions of the distribution of mass, momentum, and energy
- Mechanism(s) which damp and/or dissipate the cascade
Open Questions for Solar Probe Plus

SPP seeks to:

- Trace the flow of energy that heats the solar corona and accelerates the solar wind.
- Determine the structure and dynamics of the plasma and magnetic fields at the sources of the solar wind.
- Explore mechanisms that accelerate and transport energetic particles.

Turbulence Studies Focus on:

- Scale dependent nature of nonlinear interactions
- Statistical descriptions of the distribution of mass, momentum, and energy
- Mechanism(s) which damp and/or dissipate the cascade

Fox et al 2015
An Aside: Damping vs. Dissipation

Distinguish between:

- **Damping**
  (Energy transfer between fields and particles)
- **Dissipation**
  (Thermalization via entropy production)

Schekochihin et al 2016 has constructed a peculiar system in which the two mechanisms are decoupled, and 'anti-phase mixing' transports energy to larger, rather than smaller, velocity space structure.

Systems with infrequent Coulomb Collisions provide an opportunity to observe structures of damping before dissipation occurs.
**Mechanism(s) Damp Collisionless Turbulence**

Table 2 The five categories of models of solar wind heating have distinguishing signatures that SWEAP is designed to detect unambiguously

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Ion cyclotron(^a)</th>
<th>Turbulent cascade(^b)</th>
<th>Shock steepened acoustic modes(^c)</th>
<th>Reconnection and nano-flares(^d)</th>
<th>Filtration(^e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
<td>Ion-cyclotron wave damping</td>
<td>Turbulent cascade, spectral coupling, and dissipation</td>
<td>Compressive waves, wave steepening, shock dissipation</td>
<td>Large and small-scale reconnection, convective motions</td>
<td>Kappa distributions</td>
</tr>
<tr>
<td>Signatures in the solar wind</td>
<td>high frequency fluctuations at ion-cyclotron frequency, i+ anisotropy (T_\perp &gt; T_{</td>
<td></td>
<td>})</td>
<td>low-frequency MHD and kinetic Alfvén waves, spectral breaks, dissipation range</td>
<td>Ion beams, compressible low-frequency waves and shocks</td>
</tr>
<tr>
<td>Measurement requirement</td>
<td>p+ flow angles, 2D i+ and e-VDF at 64 Hz cross-correlated with E and B power spectra</td>
<td>Density and velocity at 1% accuracy to detect weak shocks</td>
<td>Flow angles and i+ beams at 30 Hz</td>
<td>dE/E &lt; 5% to resolve peaked VDF, p+ and e-halo</td>
<td></td>
</tr>
</tbody>
</table>


\(^b\)Matthaeus et al. (1999), Dmitruk et al. (2002), Cranmer et al. (2007), Chandran et al. (2009)

\(^c\)Bruner (1978), Ulmschneider (1985)


Velocity Space Structure of Coherent Mechanisms

**Landau Damping**

Energy Exchange via
\[ \omega - k_{\parallel}v_{\parallel} \approx 0 \]

Voitenko & Pierrard 2013; Model
Plateau formation from parallel and perpendicular phase mixing.

**Cyclotron Damping**

\[ \omega - k_{\parallel}v_{\parallel} - \Omega_s \approx 0 \]

Cranmer 2014; Model; see also Isenberg & Vasquez 2015
Creation of \( T_\perp > T_\parallel \).
Diffusion leads to input heat $Q_\perp \approx \frac{c_1 \delta v_p^3}{\rho_p} \exp \left( -\frac{c_2 w_p}{\delta v_p} \right)$.
Velocity Space Structure of Reconnection

Energization in outflow jets and trapping

Egedal et al 2012; PIC Simulation
Measuring Collisionless Energy Transfer

- Rather than infer damping mechanisms from measurements of $f(v)$, we attempt to directly measure energy transfer between $f$ and fields.
- The energy transfer is determined by the field-particle interaction term in the Vlasov Equation.
- Our metric is constructed to be applicable to single-point measurements accessible to SPP (SWEAP and FIELDS).

Outline
- Develop Field-Particle Correlation $C_F$
- Apply to Simulations:
  - 1D1V Electrostatic ($v_p$)
  - 3D2V Gyrokinetic (AstroGK)
  - 3D3V Hybrid Vlasov-Maxwell (HVM)

Klein & Howes, 2016
We consider the 1D1V Electrostatic Vlasov-Poisson system\(^1\)

\[
\frac{\partial f_s}{\partial t} + v \frac{\partial f_s}{\partial x} - \frac{q_s}{m_s} \frac{\partial \phi}{\partial x} \frac{\partial f_s}{\partial v} = 0
\]

\[
\frac{\partial^2 \phi}{\partial x^2} = -4\pi \sum_s \int dv q_s f_s
\]

Plugging the conserved energies

\[
W_\phi = \int dx \frac{E^2}{8\pi} \text{ and } W_s = \int dx \int dv \frac{m_s v^2}{2} f_s
\]

into the Vlasov Equation yields the rate of change

\[
\frac{\partial W_s}{\partial t} = \int dx \frac{\partial}{\partial x} \int dv \frac{m_s v^3}{2} f_s(x, v, t) - \int dx \int dv \frac{q_s v^2}{2} \frac{\partial f_s(x, v, t)}{\partial v} E(x, t)
\]

\[
= \int dx j_s(x) E(x, t)
\]

\(^1\)(Klein & Howes 2016, Howes et al 2016)
• \( \frac{\partial W_s}{\partial t} \) is observationally accessible only with full knowledge of \( f_s(x, v, t) \) and \( E(x, t) \).

• \( \mathbf{j} \cdot \mathbf{E} \) describes both oscillatory and secular energy transfer between waves and particles (see Bellan 2012).

We define the phase-space energy density:
\[
w_s(x, v, t) = \frac{m_sv^2}{2} f_s(x, v, t)
\]
whose rate of change is
\[
\frac{\partial w_s(x, v, t)}{\partial t} = -m_sv^3 \frac{\partial \delta f_s}{\partial x} - \frac{q_sv^2}{2} \frac{\partial f_{0s}}{\partial v} E(x, t) - \frac{q_sv^2}{2} \frac{\partial \delta f_s}{\partial v} E(x, t).
\]

The field-particle interaction terms are responsible for secular energy transfer.
Change of Particle Energy in Phase Space

- $\partial W_s/\partial t$ is observationally accessible only with full knowledge of $f_s(x, v, t)$ and $E(x, t)$.
- $\partial W_s/\partial t$ describes both oscillatory and secular energy transfer between waves and particles.

We define the phase-space energy density:

$$w_s(x, v, t) = \frac{m_s v^2}{2} f_s(x, v, t)$$

whose rate of change is

$$\frac{\partial w_s(x, v, t)}{\partial t} = -\frac{m_s v^3}{2} \frac{\partial \delta f_s}{\partial x} - \frac{q_s v^2}{2} \frac{\partial f_{0s}}{\partial v} E(x, t) - \frac{q_s v^2}{2} \frac{\partial \delta f_s}{\partial v} E(x, t).$$

For $f_s(x, v, t) \equiv f_{0s}(v) + \delta f_s(x, v, t)$ and $f_{0s}(v) = f_{0s}(-v)$, the perturbed term is responsible secular energy transfer.

To measure secular transfer, average the field-particle term:

$$C_E(x_0, v, t_j, n_C) = \frac{1}{n_C} \sum_{i=j}^{j+n_C} -\frac{q_s v^2}{2} \frac{\partial \delta f_s(x_0, v, t_i)}{\partial v} E(x_0, t_i).$$
Using the 1D1V Vlasov-Poisson Simulation Code $\nu_p$ (Howes et al 2016, in review) we construct single point measurements of $f_e(x_0, v, t)$ and $\phi(x_0, t)$ for an initial sin wave perturbation.

$\delta f_e(x = 0, v, t)$  

$E(x = 0, t)$
The energy transfer described by the perturbed particle distribution-field product

\[-\frac{q_s v^2}{2} \frac{\partial \delta f_s(x, v, t_i)}{\partial v} E(x, t_i)\]

contains both oscillatory and secular energy components.

\[\times 10^{-3}\]
Removing Oscillatory Transfer

Averaging the product of \(-\frac{q_s v^2}{2} \frac{\partial \delta f_s}{\partial v}\) and \(E\) over time intervals \(\tau = n_C dt\) removes the oscillatory energy transform, leaving only the net secular energy transfer at a given position \(x_0\):

\[
C_E(x_0, v_0, t, \tau)
\]

\[
\int dt' C_E(x_0, v_0, t', \tau^2)
\]
By calculating $C_E(v)$, the velocity space signature of the Landau damping is recovered from a single point observation.

\[ C_E(x_0, v, t, \tau) \]

\[ \int dt' C_E(x_0, v, t', \tau) \]

We have a velocity dependent measurement of the secular transfer of energy between fields and particles using single point measurements, accessible to SPP.
What is the signature of energy transfer for Other mechanisms?

For \( F_{s0}(v) = \frac{n_s}{\sqrt{2\pi}} \exp \left[ \frac{-(v-v_{ds})^2}{2v_{ts}^2} \right] \)

we have systems unstable to fluid and kinetic instabilities\(^2\)

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\(^2\)(Klein 2016 PoP, submitted)
What is the signature of energy transfer for Other mechanisms?

We correlate

$$C_E(x, v, t_j, n_C) = -\frac{1}{n_C} \sum_{i=j}^{j+n_C} \frac{q_s v^2}{2} \frac{\partial f_s(x, v, t_i)}{\partial v} E(x, t_i).$$
What is the signature of energy transfer for Other mechanisms?
What is the signature of energy transfer for Other mechanisms at other positions?
What About **Electromagnetic**, **Turbulent** Systems?

The appropriate energy transfer for a general Vlasov Equation

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla f_s + \mathbf{a} \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0$$

Takes the form of

$$C_{\text{General}}(\mathbf{v}, t_j, \tau) = -\frac{1}{n_C} \sum_{i=j}^{j+n_C} \frac{m_s v^2}{2} \mathbf{a}(x_0, t_i) \cdot \frac{\partial f_s(x_0, \mathbf{v}, t_i)}{\partial \mathbf{v}}.$$ 

For $\mathbf{a} = \frac{q_s}{m_s} \left[ \mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right]$, the $\mathbf{v} \times \mathbf{B}$ term does no work and the transfer mediated by the electric field can be expressed as

$$C_{E_l}(\mathbf{v}, t, \tau) = -\frac{1}{n_C} \sum_{i=j}^{j+n_C} \frac{q_s v_l^2}{2} \frac{\partial f_s(x_0, \mathbf{v}, t_i)}{\partial v_l} E_l.$$
To assess the effects of the magnetic mirror force, 
\[ F_{\text{mir}} = -\mu \nabla_\parallel |B| = -\frac{m_s v_\perp^2}{2B} \nabla_\parallel |B| \] we split the Lorentz term into 
\[ \frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla f_s + \left[ \mathbf{a}_L - \frac{\mu}{m_s} \nabla_\parallel \delta B_\parallel \right] \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0 \]
where \( \mathbf{a}_L \) contains the “non-mirror” electromagnetic contribution.

The secular energy transfer due the mirror force can thus be written as:

\[ C_{B\parallel} (\mathbf{v}, t, \tau) = \frac{1}{n_C} \sum_{i=j}^{i+n_C} \frac{v_\parallel^2}{2} \left[ \frac{m_s v_\perp^2}{2B} \right] \frac{\partial f_s (\mathbf{r}_0, \mathbf{v}, t)}{\partial v_\parallel} \frac{\partial \delta B_\parallel (\mathbf{r}_0, t)}{\partial s_\parallel}. \]

We thus have correlations which capture the transfer of energy due to Landau, \( (C_{E\parallel}) \), transit time \( (C_{B\parallel}) \), and cyclotron \( (C_{E\perp}) \) damping.
Gyrokinetic Turbulence

- Gyrokinetics is a rigorous, low-frequency approximation of the full Vlasov description (Frieman & Chen, 1982) which is derived by averaging over particle’s gyromotion.
- We consider gyrokinetics for a simplified slab geometry (Howes et al 2006) using the AstroGK numerical simulation code (Numata et al 2010).

- Drive 3 simulations w/ $\beta_p = 0.3, 1.0, 3.0$.
- $k_\perp \rho_p \in [0.25, 5.5]$.
- The perturbed distribution and fields are output at select spatial points.
The $n = 0$ resonance accessible to low-frequency turbulence corresponds to Landau (Landau 1946) and Transit Time (Barnes 1966) Damping.

Proton damping rates peak for scales with non-dispersive frequencies, yielding expected peak energization to be near $v_{res} \approx \frac{v_{ts}}{\sqrt{\beta_p}}$. 
The $n = 0$ resonance accessible to low-frequency turbulence corresponds to Landau (Landau 1946) and Transit Time (Barnes 1966) Damping.

Proton damping rates peak for scales with non-dispersive frequencies, yielding expected peak energization to be near

$$v_{\text{res}} \approx \frac{v_{ts}}{\sqrt{\beta_p}}.$$
For a strongly driven turbulent gyrokinetic system, we calculate $C_{E\parallel}$ and $C_{B\parallel}$ (Klein et al 2016; in prep):

Landau and transit time damping energize different regions of phase space, allowing for an observable distinction between the two mechanisms.
To track the time evolution of $C_{E\parallel}$ and $C_{B\parallel}$, we consider correlations of the reduced distribution function:

$$C_{E\parallel}^{R}(v_{\parallel}, t_i, \tau) \equiv \frac{1}{N} \sum_{j=i}^{i+N} \frac{v_{\parallel}^2}{2} \frac{\partial}{\partial v_{\parallel}} \int dv_{\perp} \left[ \delta f(v_{\parallel}, v_{\perp}, t_j) E_{\parallel}(t_j) \right]$$

$$C_{B\parallel}^{R}(v_{\parallel}, t_i, \tau) \equiv \frac{1}{N} \sum_{j=i}^{i+N} \frac{v_{\parallel}^2}{2} \frac{\partial}{\partial v_{\parallel}} \int dv_{\perp} \left[ \frac{m_s v_{\perp}^2}{2|B|} \delta f(v_{\parallel}, v_{\perp}, t_j) \hat{k}_{\parallel} \delta B_{\parallel}(t_j) \right].$$
To track the time evolution of $C_{E\parallel}$ and $C_{B\parallel}$, we consider correlations of the reduced distribution function:

- $C_{E\parallel}(\beta_p=0.3)$ (a)
- $C_{E\parallel}(\beta_p=1.0)$ (b)
- $C_{E\parallel}(\beta_p=3.0)$ (c)
- $C_{B\parallel}(\beta_p=0.3)$ (e)
- $C_{B\parallel}(\beta_p=1.0)$ (f)
- $C_{B\parallel}(\beta_p=3.0)$ (g)

Additional diagrams and plots are shown for $t_{k\parallel}V_A$.
Preliminary Results using HVM

For a decaying turbulent Hybrid Vlasov-Maxwell system (kinetic ions, fluid electrons, simulation data from Francesco Valentini):

3D-3V HVM Simulation; $C_{E\perp}(t\Omega_p=24.66)$

- $0.004$
- $0.002$
- $0.000$
- $0.000$
- $-0.002$
- $-0.004$
- $v_{\parallel}/v_{\text{tp}}$
- $v_{\perp}/v_{\text{tp}}$
A few issues that are currently being addressed:

- **Velocity Derivatives are noisy:** Consider alternative correlation: 
  \[ C' = \sum qv f E \]

- **Spacecraft measure super-Alfvénic Solar Wind:** Synthetic Spacecraft Data from Simulations

- **Actual Spacecraft Velocity Space Resolution**
  Working with SWEAP and FIELDS to prepare for correlation measurements

- **Impulsive Mechanisms (a la reconnection)?**
  Consider the choice of correlation interval

- **Diffusive Mechanisms (a la stochastic heating)?**
  Synthetic Spacecraft Data from More Simulations, may need additional correlations (a la mirror force) for other mechanisms

*Much more work is still to come before 2018...*
Take Away Message:

- Observation of local transfer of energy as a function of velocity space is feasible using simultaneous field and particle measurements.
- Such observations are accessible to current and future missions, including DSCOVR, MMS, Solar Probe Plus, and THOR and may allow observational differentiation between damping mechanisms.
For $F_{s0}(v) = \frac{n_s}{\sqrt{2\pi}} \exp \left[ -\frac{(v-v_{ds})^2}{2v_{ts}^2} \right]$ we reformulate

$$C_E(x, v, t_j, n_C) = -\frac{1}{n_C} \sum_{i=j}^{j+n_C} \frac{q_s v^2}{2} \frac{\partial f_s(x, v, t_i)}{\partial v} E(x, t_i).$$
As $\partial \delta f_s/\partial v$ can be difficult to calculate and may produce noisy results for spacecraft data, we consider the following correlation, related to $C_E$ by an integration by parts.

$$C'_E(x_0, v, t_i, \tau) \equiv \frac{1}{N} \sum_{j=i}^{i+N} q_s v \delta f_s(x_0, v, t_j) E(x_0, t_j).$$
Isn’t this just a fancy way of calculating $j \cdot E$?

- Oscillatory ($\pi/2$ out of phase)
- Secular (in phase)

Energy transfer (see Bellan 2012). Correlating over interval $\tau$ removes the oscillatory transfer.

- Velocity space structure of the energy transfer characterizes the governing physical mechanisms.
Velocity Gradients are tricky in AGK data

As $\partial/\partial v$ can be difficult to calculate and may produce noisy results for spacecraft data, we also consider $C''_E\parallel$ and $C''_B\parallel$
Normal Modes of Unstable 1D1V ES Systems

Normal Mode Solutions: \( k \lambda_{De} = 0.2 \)

- **Case 1**
- **Case 2**
- **Case 3**

**Langmuir**

**Acoustic**

\( \gamma < 0 \) 

\( \gamma > 0 \)