# Stellar Rotation Effects on the Stellar Wind

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"The rotation of a star produces interesting, but poorly understood effects,"

H.J.G.L.M. Lamers and J.P. Casinelli, *Introduction to Stellar Winds* (1999).

## Acknowledgement

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**Abstract**

We discuss the role of the *azimuthal stellar wind flow* and the associated *centrifugal* driving scenario in the *stellar-rotation braking* mechanism.

For this purpose, we make use of *Parker-Weber-Davis* (1967) *magnetohydrodynamic (MHD) stellar wind model.*

For the case when the *magnetic field* is primarily *radial* (as that *near* the surface of a star), the *Weber-Davis (1967) slow magnetosonic critical point* becomes the *Parker sonic critical point.*

The *stellar rotation* is shown to cause the *sonic critical point* to occur *lower* in the corona, so the *stellar wind* experiences a *stronger afterburner* (as in an aircraft jet engine) action in the corona.

*Stellar rotation* is shown to lead to considerably *enhanced stellar wind acceleration* even for *moderate rotators* like the *Sun*.

For *strong rotators*, the *stellar wind* experiences an *immensely enhanced* acceleration in a *narrow shell adjacent* to the star.

Thus, the *stellar rotation* leads to *tenuous* and *faster stellar wind flows* without *change* in the *mass flux*, and hence enable *protostars* and *strong rotators* to *lose* their *angular momentum quickly*.

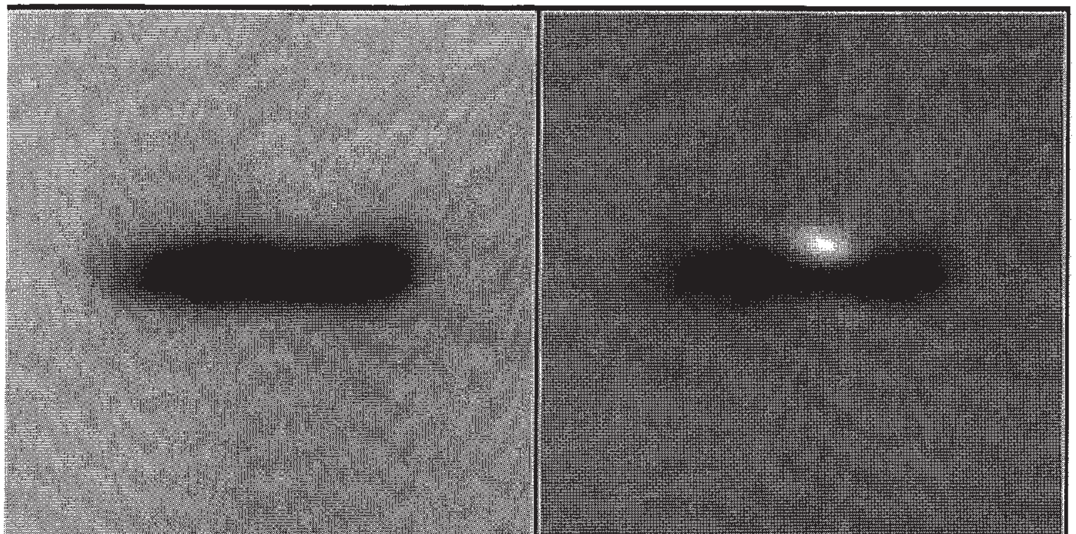
## Introduction

*Stellar wind* is an *interplanetary continuous outflow* of *hot plasma* from a star and an associated remnant of the *stellar magnetic field* that pervades the space surrounding the star (like the *heliosphere* formed by the *solar wind* ).

*Stellar winds* carry off a *very negligible* amount of *mass* from the stars while they cause (especially when the star is *magnetized*) a very effective depletion of *angular momentum* of the stars.

This is very crucial for a *protostar* because it would otherwise *breakup* due to huge *centrifugal* forces produced by the *condensation* process of the parental gas cloud.

## Solar Systems in the Making



Edge-On Protoplanetary Disk
Orion Nebula

This proplyd, Orion's largest and about 17 times Solar System's diameter, is a good example of a protostellar disk seen silhouetted against the back-lit nebula. The second image - taken through a different filter - clearly shows the hidden protostar.

Parker (1958) gave an ingenious *stationary* model for the *stellar wind*, which operates on *thermal* driving and enables the *stellar wind* to accelerate via a *continuous conversion* of the *thermal energy* in the *wind* into the *kinetic energy* of the outward *flow*.

This is predicated on the presence of a *progressively weakening retarding body force* (like *gravity*), which enables the *stellar wind* to accelerate *continuously* from *subsonic* speeds at the *coronal base* to *supersonic* speeds at *large* distances *without* requiring a physical *throat* section (as in a typical *rocket nozzle* situation).

The *corona* and the *stellar wind* are *fully ionized* and *highly electrically conducting* so the *magnetic field lines*, which are anchored into the *coronal base*, remain *frozen* in the *stellar wind*, and enforce a *co-rotation* of the *stellar wind* as if it were a *solid body* out to the *Alfvén radius* $r_A$ (where $v_r = v_{A_r} \equiv B_r/\sqrt{\rho}$) with a *larger* effective *moment of inertia* (Weber and Davis 1967).

*Parker Solar Probe* observations (Kasper et al. 2019) have indicated a *strong* coupling of the *solar wind* with *solar rotation*.

This leads to an *enhanced* outward *transfer* of *angular momentum* when the *stellar wind* eventually escapes at great distances from the star.

The *MHD* version of *Parker's (1958) hydrodynamic model* was considered by Modisette (1967), Weber and Davis (1967) and Belcher and MacGregor (1976) to treat *magnetized stellar winds*.

Here, we discuss the role of the *azimuthal wind flow* and the associated *centrifugal* driving scenario in the *stellar-rotation braking* mechanism in the *Parker-Weber-Davis MHD* model.

We show that it leads to *tenuous* and *faster stellar wind* flows, and hence enables *protostars* and *strong-rotators* to *lose* their *angular momentum quickly*.

## Weber-Davis MHD Version of Parker's Stellar Wind Model

In the *Weber-Davis (1967) MHD version of Parker's (1958) model* for the *stellar wind*, the star is assumed to have a *magnetic field* that lies in the *equatorial* plane and *depends only* on *latitude*.

This model envisages a *steady state* with complete *spherical symmetry* about the star, so the *magnetic field* and *flow variables depend only* on the *distance* $r$ from the star.

We assume that the *plasma* is *fully ionized*, *infinitely electrically conducting* and follows a *fluid* model for which the *perfect gas law* holds, and the *gas flow* occurs under *isothermal* conditions,

$$p = a^2 \rho \tag{1}$$

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*SOHO observations* (Cho et al. 2018) confirmed that the *solar wind* expands *isothermally* to considerable distances while *spacecraft measurements* (Hundhausen 1972) indicated that the *solar wind temperature drops* only by a factor of 10 from the *inner corona* to the *earth's orbit.*

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The *equation of conservation of mass,*

$$\nabla \cdot \left(\rho \mathbf{v}\right) \tag{2a}$$

implies,

$$\rho r^2 v_r = const = c_1. \tag{2b}$$

The *Gauss law* for the *magnetic field,*

$$\nabla \cdot \mathbf{B} = 0 \tag{3a}$$

gives,

$$r^2 B_r = const = c_2. \tag{3b}$$

Next, the *Ohm's law*,

$$\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B} = \mathbf{0}. \tag{4}$$

in conjunction with *Faraday's law*,

$$\nabla \times \mathbf{E} = \mathbf{0} \tag{5}$$

gives,

$$\nabla \times (\mathbf{v} \times \mathbf{B}) = \mathbf{0} \tag{6a}$$

or

$$r\left(v_\phi B_r - v_r B_\phi\right) = const = c_3. \tag{6b}$$

Near the *coronal base* anchoring the *magnetic field* lines, the *magnetic field* is primarily *radial*, we have

$$r\left(v_\phi B_r - v_r B_\phi\right) = \Omega_* r^2 B_r = \Omega_* c_2 = c_3. \tag{7a}$$

or

$$\left(v_\phi - \Omega_* r\right) B_r = v_r B_\phi \tag{7b}$$

Thus, in the *space-fixed* frame, the *magnetic field lines* exhibit a *spiral* pattern - *Parker spirals* (Parker 1958) which have been confirmed by the *magnetometer data* on *Mariner II* (Ness et al. 1966, Neugebauer and von Steiger 1966).

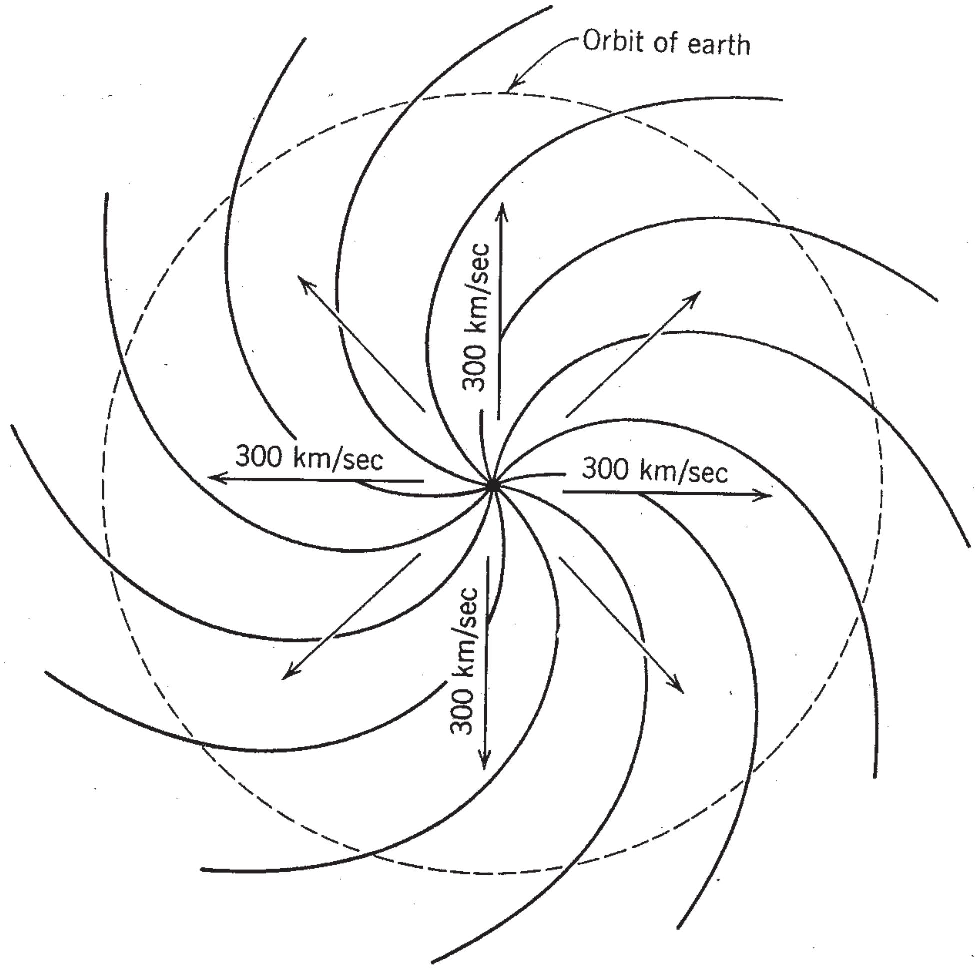


Figure 6.16. Parker spiral interplanetary magnetic field lines in the equatorial plane ($\theta = 90°$). (From Parker, 1963.) Copyright © (1963 John Wiley and Sons, Inc.). Reprinted by permission of John Wiley and Sons, Inc.

## Azimuthal Momentum Balance

Next, the *azimuthal* component of the *momentum equation*,

$$\rho\left(v_r\frac{dv_\phi}{dr}+\frac{v_r v_\phi}{r}\right)=B_r\frac{dB_\phi}{dr}+\frac{B_r B_\phi}{r} \tag{8}$$

on using (2) and (3), gives

$$r v_\phi-\frac{c_2}{c_1}rB_\phi=const=L \tag{9}$$

Here $L$ is the *total angular momentum* per unit mass carried away by the *stellar wind.*

On the other hand, using (2) and (3), (7a) and (9) lead to

$$r\,v_\phi = \frac{\Omega_* r^2 - M_{A_r}^2 L}{1 - M_{A_r}^2}, r\,B_\phi = \frac{\Omega_* r^2 - L}{(c_2/c_1)\left(1 - M_{A_r}^2\right)} \qquad \text{(10a, 11a)}$$

where,

$$M_{A_r}^2 \equiv \frac{v_r^2}{v_{A_r}^2}. \qquad (12)$$

The condition,

$$r = r_A : M_{A_r}^2 = 1 \tag{13}$$

then requires,

$$L = \Omega_* r_A^2. \tag{14}$$

(14) implies that, in the *Weber-Davis model*, the *total angular momentum* per unit mass in the *stellar wind* is determined as if the latter were in a *solid-body rotation* out to $r_A$,

$$v_\phi \approx \Omega_* r, \; r \lesssim r_A. \tag{15}$$

Thus,

$$r v_\phi = \frac{\Omega_* \left(r^2 - M_{A_r}^2 r_A^2\right)}{\left(1 - M_{A_r}^2\right)}, r B_\phi = \frac{\Omega_* \left(r^2 - r_A^2\right)}{(c_2/c_1)\left(1 - M_{A_r}^2\right)}. \qquad \text{(10b,11b)}$$

(10b) and (11b), in conjunction with (13), implies that $B_\phi < 0, \forall r$, so the *magnetic field lines* are *trailing spirals*, as expected.

**Radial Momentum Balance**

The *radial* component of the *momentum equation*,

$$\rho\left(v_r\frac{dv_r}{dr}-\frac{v_\phi^2}{r}\right)=-\frac{dp}{dr}-\frac{\rho GM}{r^2}-\frac{B_\phi}{r}\frac{d}{dr}\left(rB_\phi\right). \tag{16}$$

on using (10a) and (11a), gives (Belcher and MacGregor 1976),

$$\frac{r}{v_r}\frac{dv_r}{dr}=\frac{\left(v_r^2-v_{A_r}^2\right)\left(2a^2+v_\phi^2-GM/r\right)+2v_r v_\phi v_{A_r} v_{A_\phi}}{\left(v_r^2-v_{A_r}^2\right)\left(v_r^2-a^2\right)-v_r^2 v_{A_\phi}^2} \tag{17}$$

where,

$$v_{A_\phi}^2 \equiv \frac{B_\phi^2}{\rho},\ v_{A_r}^2 \equiv \frac{B_r^2}{\rho}.$$

Equation (17) may be rewritten as

$$\frac{r}{v_r}\frac{dv_r}{dr} = 2a^2 \frac{\left(v_r^2 - v_{A_r}^2\right)\left[r\left\{1 + \frac{1}{2a^2}\left[v_\phi^2 + \frac{2v_{A_\phi}^2\left(\frac{r^2}{r_A^2}\right)\left(1-\frac{v_r}{v_{A_r}}\right)}{\left(1-r^2/r_A^2\right)\left(1-M_{A_r}^2\right)}\right]\right\} - r_*\right]}{r\left(v_r^2 - c_+^2\right)\left(v_r^2 - c_-^2\right)} \tag{18}$$

where $c_\pm$ are, respectively, the speeds of the *magnetosonic fast* and *slow* waves propagating in the *radial* direction *away* from the star,

$$c_\pm^2 \equiv \frac{v_{A_r}^2 + v_{A_\phi}^2 + a^2}{2} \pm \sqrt{\left(\frac{v_{A_r}^2 + v_{A_\phi}^2 + a^2}{2}\right)^2 - a^2 v_{A_r}^2}. \tag{19}$$

**The Effect of Azimuthal Stellar Wind Flow**

If the *magnetic field* is close to being *radial*, (like *near* the surface of the star, so $v_{A_\phi} \approx 0$), (19) gives

$$c_+ \approx v_{A_r}, \; c_- \approx a \tag{20}$$

while (7b) gives,

$$v_\phi \approx \Omega_* r \tag{21}$$

in consistency with (15), implying the *co-rotation* of the *stellar wind* in this region.

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Observations by the *Parker Solar Probe* (Kasper et al. 2019) of the *solar wind* plasma flow show *large azimuthal* velocities (25-35 mps) at heliocentric distances of about 35 solar radii indicating a *strong* coupling of the *solar wind* with *solar rotation*.

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*Numerical calculations* of Hartmann and MacGregor (1982) confirmed (21) to hold in the region $R < r < r_A$, while *numerical calculations* of Belcher and MacGregor (1976) showed that $B_\phi$ becomes non-negligible in the region $r > r_s$, where $r = r_s : v_r = c_-$.

Using (20), equation (18) becomes

$$\frac{1}{v_r}\frac{dv_r}{dr} \approx \frac{2a^2}{r^2}\frac{\left[r\left(1+v_\phi^2/2a^2\right)-r_*\right]}{\left(v_r^2-a^2\right)}. \tag{22}$$

Equation (22) amounts to *Parker's stellar wind* equation effectively modified to include *azimuthal wind flow* within the *hydrodynamic* framework.

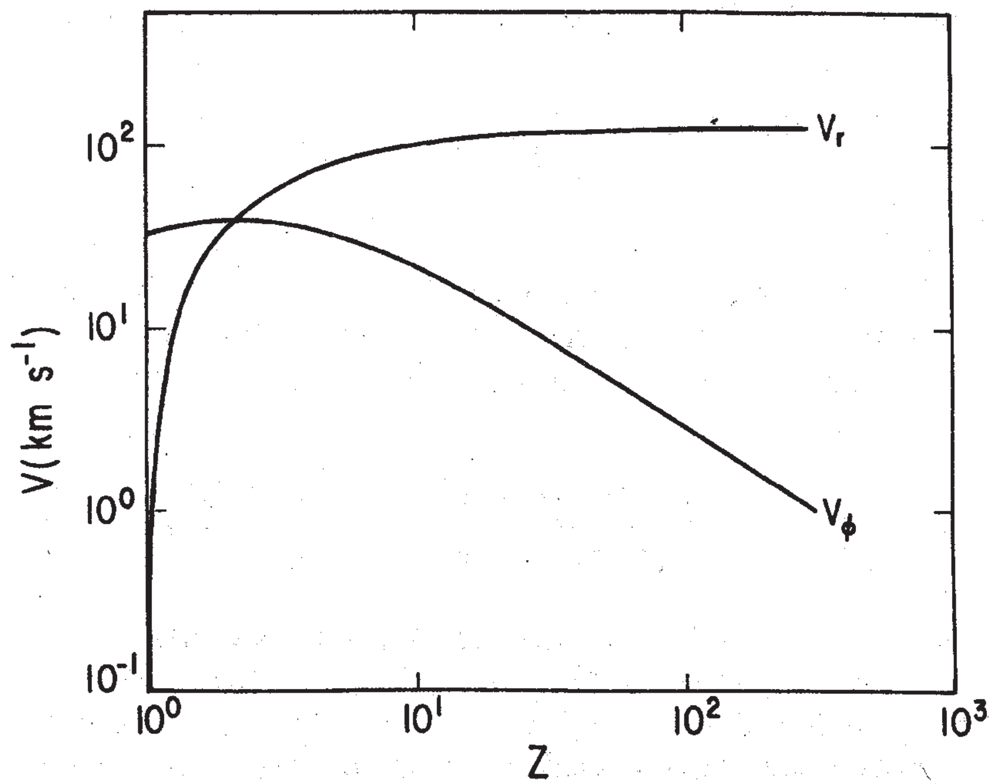


FIG. 2.—Radial and azimuthal velocities as functions of distance from the reference level for the protostellar wind solution corresponding to $\alpha = 0.9$.

(From Hartmann & MacGregor 1982)

Using (21), equation (22) becomes (as also given by Hartmann and MacGregor 1982),

$$\frac{1}{v_r}\frac{dv_r}{dr} \approx \frac{2a^2}{r^2}\frac{\left(r - r_* + \Omega_*^2 r^3/2a^2\right)}{\left(v_r^2 - a^2\right)}, r \lesssim r_A. \tag{23a}$$

Here $r_*$ locates the *sonic critical point* in *Parker's hydrodynamic model*,

$$r_* \equiv \frac{GM}{2a^2}. \tag{24a}$$

Equation (23a) implies that *MHD* effects may be viewed to be included in *Parker's hydrodynamic model*, in the *first* approximation, via a *co-rotation* of the *stellar wind* out to $r_A$.

**The Parker Sonic Critical Point.**

Equation (23a) may be rewritten as

$$\frac{1}{v_r}\frac{dv_r}{d\xi} = \frac{\Omega_*^2 \tilde{r}^2 \left(\xi^3 + \gamma\xi - 1\right)}{\xi^2 \left(v_r^2 - a^2\right)} \tag{23b}$$

or

$$\frac{1}{v_r}\frac{dv_r}{d\xi} = \frac{\Omega_*^2 \tilde{r}^2 \left[\xi - (\alpha - \beta)\right] \left[\left(\xi + \frac{\alpha - \beta}{2}\right)^2 + \frac{3}{4}(\alpha + \beta)^2\right]}{\xi^2 \left(v_r^2 - a^2\right)} \tag{23c}$$

Here,

$$\xi \equiv \frac{r}{\tilde{r}}, \tilde{r} \equiv \left(\frac{GM}{\Omega_*^2}\right)^{1/3}$$
$$\alpha, \beta \equiv \left[\sqrt{\frac{1}{4} + \frac{\gamma^3}{27}} \pm \frac{1}{2}\right]^{1/3}, \gamma \equiv \frac{\tilde{r}}{r_*} = 2\left(\frac{a^3}{GM\Omega_*}\right)^{2/3}. \tag{24b}$$

Thus, the *Parker sonic critical point* in a *stellar wind* with *azimuthal flow* is given by

$$r = \hat{r} \equiv (\alpha - \beta)\, \tilde{r} : v_r = a \tag{25a}$$

where the numerator and denominator in equation (23c) vanish *simultaneously*, so equation (23a) leads to a physically acceptable *smooth* solution.

The *cubic* equation,

$$\xi^3 + \gamma\xi - 1 = 0$$

has one *real* root $(\alpha - \beta)$ and two *complex conjugate* roots (see (23c)).

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The *sonic critical point* may be viewed as an approximation to the *slow magnetosonic critical point* of the *Weber-Davis MHD formulation.*

For *strong rotators*, ($\gamma << 1$), (25a) gives

$$\hat{r} \approx \left[1 - \left(\frac{\gamma}{3}\right)^{1/3} + 0\left(\frac{\gamma}{3}\right)^{2/3}\right]\tilde{r}$$

or

$$\hat{r} \approx \left(1 - \frac{\tilde{r}}{3r_*}\right)\tilde{r} \approx \tilde{r},\ \frac{\tilde{r}}{r_*} << 1. \tag{25b}$$

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*Strong rotators* appear to be indeed a better system than *moderate* (or *slow*) *rotators* for the application of the *Weber-Davis model* because the *observed flattening* of *strong rotators* facilitates the *confinement* of *mass* and *angular momentum loss* to the *equatorial* plane.
Some *strong rotators* in the *main-sequence* stars are,

* *Altair* (rotation speed 432,000 mph),
* *Regulus A* (rotation speed 700,000 mph),
* *Achernar* (rotation speed 558,000 mph),
* *VFTS 102* (rotation speed 1 million mph) in the *Large Magellanic cloud.*

*Achernar* is an *oblate spheroid* with an *equatorial* diameter 56% greater than its *polar* diameter.

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So, for *strong rotators*, the *sonic critical point* (25a) is essentially determined by the basic stellar parameters like,

* the *mass* $M$,
* the *angular velocity* $\Omega_*$.

This signifies the dominance of *centrifugal* and *magnetic* drivings in accelerating the *stellar wind* for *strong rotators*, (*thermal* driving dominates for *solar-type slow rotators*).

For *slow rotators* ($\gamma >> 1$), (25a) gives

$$\hat{r} \approx \left[1 - \frac{\sqrt{3}}{2}\left(\frac{r_*}{\tilde{r}}\right)^{3/2}\right] r_*, \ \frac{\tilde{r}}{r_*} >> 1. \qquad (25c)$$

For the *Sun*, which is a *moderate rotator*, ($\Omega_* \approx 2{\cdot}8 \times 10^{-6} rad/sec$), $\gamma \approx 1 \cdot 8$, we have

$$\hat{r} \approx {\cdot}49\tilde{r} \approx {\cdot}87 r_*.$$

So, the *sonic critical point* (25a) for the *Sun*, upon including the *azimuthal flow* in the *solar wind*, occurs *lower* in the *corona* than the *sonic critical point* in *Parker's hydrodynamic model.*

This has been confirmed by the *numerical simulation* of *stellar winds* (Keppens and Goedbloed 1999).

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As a consequence of this, the *stellar wind* also experiences a stronger *afterburner* (as in an aircraft jet engine (Parker 1958)) action in the *corona* and turns out to be *tenuous* and *fast* without changing the mass flux signifying the conversion of *rotational* energy of the star into *streaming* energy of the *stellar wind*.

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This is consistent with the *numerical results* of Belcher and MacGregor (1976) showing that the *wind velocity* $v_r$ *increases* as $\Omega_*$ *increases*.

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**The Analytical Solution .**

The solution of equation (23a), upon imposing the *flow smoothness* condition (25a), is given by

$$\frac{v_r^2}{a^2} - \log\left(\frac{v_r^2}{a^2}\right) = 4\log\left(\frac{r}{\hat{r}}\right) + 4\frac{r_*}{r} + \frac{\Omega_*^2 r^2}{a^2} - \left(4\frac{r_*}{\hat{r}} + \frac{\Omega_*^2 \hat{r}^2}{a^2} - 1\right) \quad (26a)$$

or

$$v_r e^{-\left(v_r^2/2a^2\right)} = a\left(\frac{\hat{r}}{r}\right)^2 e^{\left[\frac{2r_*}{\hat{r}}\left(1-\frac{\hat{r}}{r}\right) + \frac{\Omega_*^2 \hat{r}^2}{2a^2}\left(1-\frac{r^2}{\hat{r}^2}\right) - \frac{1}{2}\right]} \quad (26b)$$

The solution (26) is plotted in Figure 1, which shows considerably *enhanced* acceleration of the *stellar wind* caused by the *stellar rotation* for even a *moderate rotator* like the *Sun*, $(\gamma \approx 1 \cdot 8)$.

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*Observational data* (Belcher and MacGregor 1976) shows that, for the *Sun* (rotation rate $3 \times 10^{-6}$ rad/sec), the *loss* of *rotational* kinetic energy is $2 \times 10^{25}$ ergs/sec, while for a typical *main-sequence* faster rotating star (rotation rate $3 \times 10^{-5}$ rad/sec) the *loss* of *rotational* kinetic energy is larger and is $2 \times 10^{29}$ ergs/sec.

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Figure 1 also confirms that the *Parker sonic critical point* for the *Sun*, upon including the *azimuthal flow* in the *solar wind*, occurs *lower* in the *corona* than the *critical point* in *Parker's hydrodynamic model.*

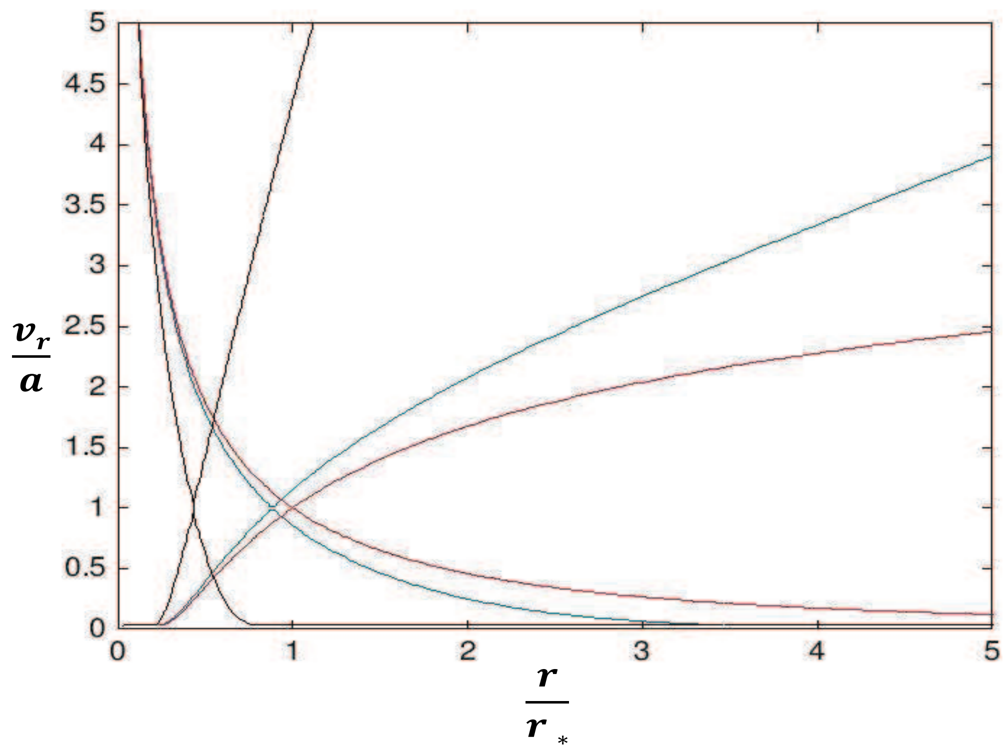


**Figure 1.** Effect of stellar rotation on the stellar wind speed – (i) Parker model $\gamma = \infty$ (red), (ii) Weber-Davis model with co-rotating solar wind with $\gamma = 1.8$ (blue), (iii) Weber-Davis model with co-rotating stellar wind for a strong rotator with $\gamma = 0.427$ (black).

**Strong Rotators.**

For *strong rotators* $(\gamma << 1)$, equation (23a) becomes

$$\left( v_r - \frac{a^2}{v_r} \right) \frac{dv_r}{dr} \approx \Omega_*^2 \left( r - \frac{\tilde{r}^3}{r^2} \right). \tag{27}$$

Upon imposing the *flow smoothness* condition (25a), equation (27) gives

$$\frac{v_r^2}{a^2} - \log\left( \frac{v_r^2}{a^2} \right) \approx \frac{\Omega_*^2}{a^2} \left( r^2 + \frac{2\tilde{r}^3}{r} - 3\tilde{r}^2 \right) + 1 \tag{28a}$$

or

$$\left(\frac{v_r}{a}\right) e^{-\left(v_r^2/2a^2\right)} \approx e^{-\left[\frac{\Omega_*^2}{2a^2}\left(r^2+\frac{2\tilde{r}^3}{r}-3\tilde{r}^2\right)+1\right]}. \tag{28b}$$

For $r << \tilde{r}$, we obtain

$$v_r \approx ae^{-(2r_*/r)}. \tag{29}$$

(29) implies that, for *strong rotators* most of *stellar wind acceleration*, occurs in a *narrow shell close* to the star.

This, in conjunction with (26), is consistent with the *numerical results* of Hartmann and MacGregor (1982) showing that the *stellar wind* becomes *super-Alfvénic lower* in the *corona* for *stronger stellar rotation*.

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The solution (28) is plotted in Figure 1, and shows, for *strong rotators*, thanks to *centrifugal* driving, an immensely *enhanced* acceleration of the *stellar wind* occurring in a *narrow shell adjacent* to the star,

This is underscored by the *sonic critical point*, for *strong rotators*, occurring considerably *lower* in the *corona*, as shown by Figure 1, hence supporting a huge *afterburner* action in the *corona*.

**Discussion**

Parker (1958) made an ingenious demonstration that a *stellar wind*, thanks to a *progressively weakening retarding body force* (like *gravity*), can evolve from *subsonic* to *supersonic* speeds even in a purely *divergent* channel (like the *interplanetary space*).

In view of the *dominance* of the *magnetization effects* over *thermal* effects away from the star (even for the case of a *weakly-magnetized* star like the *Sun*), the *interaction* of the *stellar magnetic field* with the *expanding corona* attains great significance.

We have therefore considered the *Weber-Davis (1967) MHD version* of *Parker's (1958) hydrodynamic model* with the aim of clarifying the role of the *azimuthal wind flow* in the *stellar-rotation braking* mechanism.

The *stellar rotation* is shown to cause the *sonic critical point* to occur *lower* in the *corona*, so the *stellar wind* experiences a *stronger afterburner* action in the *corona*, and considerably *enhanced* acceleration even for *moderate rotators* like the *Sun*.

The *stellar wind*, for *strong rotators*, thanks to *centrifugal* driving, is shown to experience an *immensely enhanced acceleration* in a *narrow shell adjacent* to the star.

For *strong rotators*, the *sonic critical point* is shown further to be determined only by the basic stellar parameters like the *mass* $M$ and the *angular velocity* $\Omega_*$, which signifies the *dominace* of *centrifugal* and *magnetic drivings* in *accelerating* the *stellar wind* for such stars.

The *stellar rotation* leads to *tenuous* and *faster stellar wind* flows without change in the *mass flux*, and hence provides an *efficient physical mechanism* for *protostars* and *strong rotators* to *lose* their *angular momentum* quickly.

Finally, it may be noted that the *distances* of the *stellar objects* and the *difficulty* in accessing even the *solar coronal base* conditions along with the *simplifying approximations* implicit in the *theoretical formulations* render the *observation* at this time *inadequate* to fully *corroborate* a *particular proposed acceleration mechanism* for the *stellar wind.*

## Where do We Go from Here

My present theoretical formulation has a modest aim at providing a minor qualitative step beyond *Parker's* original ingenious analytically tractable theoretical model, namely, include the effect of *stellar rotation* albeit in a first-order approximation via *co-rotation* ansatz.

So the resulting formulation has the merit of being still analytically tractable and serve as a qualitative guide on the general principles and try to provide some physical insights.

In order to make this simple model conform better with reality, there are several issues that I am currently addressing,

* *stability* of the *stellar wind solutions*, (preprint: arXiv.2007.06545, (2020))

* extension to *polytropic stellar winds*, (preprint: arXiv:. 1905.03630, (2020))

There are several topics addressing relaxation of several restrictions,

* *co-rotation* of the *stellar wind*,

* pressure *isotropy*,

* *monopole* magnetic field distributed only in the *equatorial* plane,
* *spherical symmetric* geometry,
* ignoring *coronal heating* details

that need to be considered.